Resistivity

\[ R = \rho \frac{L}{H \cdot W} \]

\( \rho \) = resistivity of the material (\( \text{\(\Omega\)-m})

\( \rho_{\text{Al}} = 2.7 \times 10^{-8} \text{\(\Omega\)-m} \)

\( \rho_{\text{Cu}} = 1.7 \times 10^{-8} \text{\(\Omega\)-m} \)

\[ R_\square = \frac{\rho}{H} \quad \text{"\(\Omega\) per square"} \]

\[ R = R_\square \frac{L}{W} = R_\square \cdot \text{"Num of squares"} \]

Skin Effect

High-freq. currents stay near the surface of a conductor \( \rightarrow \) Resistance depends on the signal frequency

\[ \delta = \text{skin depth} = \sqrt{\frac{\rho}{\pi f \mu}} \]

\[ \delta = K \sqrt{\frac{1}{f}} \]

\( f \) = freq of signal

\( \mu \) = permeability of surrounding dielectric
Inductance
- Difficult to calculate
- Intentional
  - Analog
  - RF
- Unintentional
  - High-speed signals on/off chip
  - Power grids

To find inductance
1) Complex 3-D field solver
2) Measurement \[ \Delta V = L \frac{di}{dt} \]
   \[ L = \frac{\Delta V}{\frac{di}{dt}} \]
3) \[ cl = \varepsilon M \]
   \[ l = \frac{\varepsilon M}{c} \]
wire models

1) actual
   - distributed $R, C, L$

2) ideal
   - $R=0$, $L=0$, $C=0$
   - single node

3) lumped
   - distributed parameters placed in one spot

   a) lumped $C$ (tcsim)
      good for: low $R$ wires
      low frequencies

   b) lumped $RC$
      - consider wire tree and not just
point-to-point
- substitute an $R$ and $C$ for each segment
- requires differential equations

- Use Elmore Delay model
  - trees (no loops)
  - single input
  - all caps to ground

- "path resistance": all $R$s along a path
  Ex: $s \rightarrow t$: $R_1 + R_3 + R_4$
- "shored path resistance" = $R_{ik}$
  resistance shored among paths $s \rightarrow i$, $s \rightarrow k$
  Ex: $R_{46} = R_1 + R_3$
  $R_{26} = R_1$
  $R_{ik} = \sum R_j \Rightarrow (R_j \in [\text{path}(s \rightarrow i) \cap \text{path}(s \rightarrow k)])$
Elmore delay = First-order time constant
= Dominant time constant
= Approx. actual delay

Elmore delay at node 4
\[ \tau_{D4} = \sum_{k=1}^{N=6} c_k R_{4k} \]

\[ = c_1 R_1 + c_2 R_1 + c_3 (R_1 + R_3) + c_4 (R_1 + R_3 + R_4) + c_5 (R_1 + R_3) + c_6 (R_1 + R_3) \]

- Considers all caps
- RC of unshared paths is not accounted for

**Entire model:** lumped RC - cont'd

\[ V_{in} \rightarrow \frac{1}{RC} \rightarrow \frac{V_{out}}{V_{dd}} \]

- \( V_{in} \): 0 \( \rightarrow \) 0.1 V step
- \( V_{out} \): 0 \( \rightarrow \) 0.9 V

\[ t_p = 0.69 \cdot R \cdot C \]

Delay is too large because entire cap is charged through entire resistance
2) Split up into N pieces:

\[ V_{\text{in}} \to V_{\text{in}} \to \ldots \to V_{\text{out}} \]

where \( R_i = \frac{R_{\text{total}}}{N} \)
\( C_i = \frac{C_{\text{total}}}{N} \)

For large \( N \), using Elmore delay equation,

\[ T_{\text{DN}} = \frac{RC}{2} = \frac{R_{\text{total}} C_{\text{total}}}{2} \]

- wire only, no \( C_2 \)
- \( R = r \cdot L \)
- \( C = c \cdot L \)

Then \( T_{\text{DN}} = \frac{r c L^2}{2} \)

delay increases with \( L^2 \)!

3) Distributed RC line

Using P.D.E.

\[ RC \frac{\partial V}{\partial t} = \frac{\partial^2 V}{\partial x^2} \]

\[ V: \ 0 \to \frac{V_{\text{dd}}}{2} \text{ at end of wire} \]

\[ t_p = 0.38 \ RC = 0.38 \ r c L^2 \]

- distributed RC
- not \( C_2 \)
<table>
<thead>
<tr>
<th>Input to wire</th>
<th>Output of wire</th>
<th>Lumped RC</th>
<th>Distrib RC</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 0 \to V_{DD}/2 )</td>
<td>0.69 RC</td>
<td>0.38 RC</td>
<td></td>
</tr>
<tr>
<td>( 0 \to 63% (T) )</td>
<td>RC</td>
<td>0.5 RC</td>
<td></td>
</tr>
<tr>
<td>( 10% \to 90% (t_r) )</td>
<td>2.2 RC</td>
<td>0.9 RC</td>
<td></td>
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</tbody>
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