2015 IEEE 54th Annual Conference on Decision and Control (CDC)
December 15-18, 2015. Osaka, Japan

Controllers for interconnected systems with communication delays

A. N. Gündeş and Liansing C. Chow

***Abstract*— Finite-dimensional controllers are developed for interconnected systems, where arbitrary time delays occur in the transmission loops. Various cascade or feedback interconnections are considered, with time delay terms appearing in numerator or denominator matrices of the transfer-functions of the plants to be controlled. The proposed designs achieve low-order and integral-action controllers. The methods are applied to load frequency control of single-area and multi-area interconnections of power systems.**

I. Introduction

Many control applications involve information exchange of control and feedback signals over communication networks in the form of data packages. Applications that include data networks in a control loop also introduce network-induced delay effect in data transfers between the controller and the remote system. A continuous-time signal transmitted over a network is sampled and digitally encoded; the data transmitted over the network is decoded at the receiver. The overall delay between sampling and decoding include network access delays due to a shared network's data acceptance time, and transmission delays while data is in transit inside the network depending on congestion and channel quality. Time delays may destabilize the system and degrade performance, and their effects cannot be ignored in control applications [7]. Stability of systems subject to time delays has been investigated extensively. Several delay-independent and delay-dependent stability results were obtained. For some linear, time-invariant (LTI) plant classes and fractional-order systems subject to time delays, proportional-integral-derivative (PID) controllers were proposed [1], [2], [5], [9], [3], [8]. This work presents finite-dimensional stabilizing controller synthesis methods for LTI, single-input single-output (SISO) or multi-input multi-output (MIMO) system interconnections subject to time delays. The proposed low-order controllers are simple, and in most cases they provide integral-action so that step-input references are tracked asymptotically with zero steady-state error. Delay terms may appear in various different parts of a system's mathematical description. Systems that are subject to input and output delays contain time delay terms in the numerators of their transfer-functions; cascade interconnections of systems with delayed information transfer between components are in this category of systems. Section III is devoted to finite-dimensional controller design for these systems, with the assumption that the poles of the delay-free parts of the plants are at the origin and the open left-half complex-plane, and their zeros are completely unrestricted. Some systems have delay terms in the denominators of their transfer-functions as in the case of feedback interconnections with transmission delays. Section IV deals with designs under the assumption that the zeros are either at infinity or the open left-half plane, but the poles are completely unrestricted. The proposed design methods are applied to load frequency control of single-area and multi-area power systems in Section V [4].

The authors are with the Department of Electrical and Computer Engineering, University of California, Davis, CA 95616. angundes@ucdavis.edu, lschow@ucdavis.edu

The notation used here is standard: $\mathbb{C}, \mathbb{R}, \mathbb{R}_+$ denote complex, real, and positive real numbers. The extended closed right-half complex plane is $\mathcal{U} = \{s \in \mathbb{C} \mid \mathcal{R}e(s) \geq 0\} \cup \{\infty\}$; $\mathbb{C}_-$ is the open left-half complex-plane. The set of real proper rational functions (of s) is $\mathbf{R_p}$; $\mathbf{S} \subset \mathbf{R_p}$ is the stable subset with no poles in $\mathcal{U}$; $\mathcal{M}(\mathbf{S})$ is the set of matrices with entries in $\mathbf{S}$. The space $\mathcal{H}_\infty$ is the set of all bounded analytic functions in $\mathbb{C}_+$. We drop (s) in transfer-matrices such as $G(s)$ when this is clear from the context. The $(m \times m)$ diagonal matrix whose diagonal entries are $a_1, \ldots, a_m$ is diag $[a_1, \cdots, a_k]$. The $(m \times m)$ identity matrix is I_m. For $h \in \mathcal{H}_\infty$, the norm is defined as $\|h\|_\infty = \text{ess sup}_{s \in \mathbb{C}_+} |h(s)|$; ess sup is the essential supremum. Since all norms of interest here are $\mathcal{H}_\infty$ norms, the subscript is dropped, i.e., $\|\cdot\|_\infty \equiv \|\cdot\|$. A matrix-valued function H is in $\mathcal{M}(\mathcal{H}_\infty)$ if all its entries are in $\mathcal{H}_\infty$; $\|H\|_\infty = \text{ess sup}_{s \in \mathbb{C}_+} \overline{\sigma}(H(s))$, where $\bar{\sigma}$ denotes the maximum singular value. A system whose transfer-matrix is H is stable if $H \in \mathcal{M}(\mathcal{H}_\infty)$. A square $H \in \mathcal{M}(\mathcal{H}_\infty)$ is unimodular if $H^{-1} \in \mathcal{M}(\mathcal{H}_\infty)$. For $G \in \mathbf{R_p}^{m \times m}$, coprime factorizations over $\mathbf{S}$ are used; i.e., $G = Y^{-1}X$ is a left-coprime factorization (LCF), $G = \widetilde{X}\widetilde{Y}^{-1}$ is a right-coprime factorization (RCF), $X, Y, \widetilde{X}, \widetilde{Y} \in \mathbf{S}^{m \times m}$, $\det Y(\infty) \neq 0$, and $\det \widetilde{Y}(\infty) \neq 0$. For the delayed plant, coprime factorizations over $\mathcal{H}_\infty$ are used; i.e., $\mathcal{G} = \mathcal{Y}^{-1}\mathcal{X}$ is an LCF, $\mathcal{G} = \widetilde{\mathcal{X}}\widetilde{\mathcal{Y}}^{-1}$ is an RCF, $\mathcal{X}, \mathcal{Y}, \widetilde{\mathcal{X}}, \widetilde{\mathcal{Y}} \in \mathcal{H}_\infty{}^{m \times m}$. Let $\mathcal{G} = \mathcal{Y}^{-1}\mathcal{X} = \widetilde{\mathcal{X}}\widetilde{\mathcal{Y}}^{-1}$ have full normal-rank m. The transmission-zeros of $\mathcal{G} = \mathcal{Y}^{-1}\mathcal{X} = \widetilde{\mathcal{X}}\widetilde{\mathcal{Y}}^{-1}$ in $\mathcal{U}$ are $s_o \in \mathcal{U}$ such that rank$\mathcal{X}(s_o) < m$, equivalently, rank$\widetilde{\mathcal{X}}(s_o) < m$. The blocking-zeros of $\mathcal{G}$ in $\mathcal{U}$ are $s_o \in \mathcal{U}$ such that $\mathcal{G}(s_o) = 0$, equivalently, $\mathcal{X}(s_o) = \widetilde{\mathcal{X}}(s_o) = 0$. Blocking-zeros of $\mathcal{G}$ are also transmission-zeros. These zeros in $\mathcal{U}$ are called $\mathcal{U}$-zeros when a distinction between transmission-zeros and blocking-zeros is not crucial.

II. Problem Description

Consider the feedback system $Sys(\mathcal{G}, C)$ in Fig. 1; $\mathcal{G}$ is the plant's transfer-function with time delays, $C \in \mathbf{R_p}^{m \times m}$ is the finite-dimensional controller's transfer-function. It is assumed that the feedback system is well-posed, and the plant and the controller have no unstable hidden-modes. With r, w as input, x, y as output vectors, the

978-1-4799-7885-4/15/$31.00 ©2015 IEEE

closed-loop map $\mathcal{W}$ from (r,w) to (x,y) is $\mathcal{W} = \begin{bmatrix} C(I+\mathcal{G}C)^{-1} & -C(I+\mathcal{G}C)^{-1}\mathcal{G} \\ \mathcal{G}C(I+\mathcal{G}C)^{-1} & (I+\mathcal{G}C)^{-1}\mathcal{G} \end{bmatrix}$. The input-output map from r to y is $H_{yr} = \mathcal{G}C(I+\mathcal{G}C)^{-1}$; the input-error map from r to e is $H_{er} = I - H_{yr} = (I+\mathcal{G}C)^{-1}$.

Definition 1: **a)** The system $Sys(\mathcal{G}, C)$ in Fig. 1 is stable if $\mathcal{W} \in \mathcal{M}(\mathcal{H}_\infty)$. **b)** $Sys(\mathcal{G}, C)$ is stable and has integral-action if $\mathcal{W} \in \mathcal{M}(\mathcal{H}_\infty)$, and $H_{er}(0) = 0$. **c)** C is a stabilizing controller if C is proper and $H \in \mathcal{M}(\mathcal{H}_\infty)$. **d)** C is an integral-action controller if C is a stabilizing controller, and $\widetilde{D}(0) = 0$ for any RCF $C = \widetilde{N}\widetilde{D}^{-1}$. □

Fact 1: Let $\mathcal{G} = \mathcal{Y}^{-1}\mathcal{X}$ be an LCF, $\mathcal{G} = \widetilde{\mathcal{X}}\widetilde{\mathcal{Y}}^{-1}$ be an RCF of the plant $\mathcal{G}$. Let $C(s) = D^{-1}N$ be an LCF, and $C(s) = \widetilde{N}\widetilde{D}^{-1}$ be an RCF of the controller C. Define $M \in \mathcal{M}(\mathcal{H}_\infty)$ and $\widetilde{M} \in \mathcal{M}(\mathcal{H}_\infty)$ as in (1):

$$M := \mathcal{Y}\,\widetilde{D} + \mathcal{X}\,\widetilde{N}\ , \quad \widetilde{M} := D\,\widetilde{\mathcal{Y}} + N\,\widetilde{\mathcal{X}}\ . \tag{1}$$

Then the feedback system $Sys(\mathcal{G}, C)$ in Fig. 1 is stable if and only if $M^{-1} \in \mathcal{M}(\mathcal{H}_\infty)$, equivalently, $\widetilde{M}^{-1} \in \mathcal{M}(\mathcal{H}_\infty)$. □

Let $Sys(\mathcal{G}, C)$ in Fig. 1 be stable. Let $r(t) = K1(t)$, $K \in \mathbb{R}^m$, and $1(t)$ is the unit step function. The steady-state error $e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} H_{er}(s)R(s) = H_{er}(0)K$ is zero for all $K \in \mathbb{R}^m$ if and only if $H_{er}(0) = 0$. By Definition 1-(b), the stable system achieves asymptotic tracking of constant reference inputs with zero steady-state error if and only if it has integral-action. By (1), $H_{er} = \widetilde{D}M^{-1}\mathcal{Y} = (I - \widetilde{\mathcal{X}}\widetilde{M}^{-1}N)$. By Definition 1-b, the system has integral-action if $C = \widetilde{N}\widetilde{D}^{-1}$ is an integral-action controller since $\widetilde{D}(0) = 0$ implies $H_{er}(0) = (\widetilde{D}M^{-1}\mathcal{Y})(0) = 0$. If $\mathcal{G} \in \mathcal{M}(\mathcal{H}_\infty)$, then the system has integral-action if and *only if* C is an integral-action controller. If $\mathcal{Y}(0) = 0$, then integral-action is ensured for the system, but it is included in the controller for robustness by the internal model principle.

Fact 2: If the stable system $Sys(\mathcal{G}, C)$ has integral-action, then $\mathcal{G}$ has no transmission-zeros at $s = 0$. □

Due to the necessary condition in Fact 2 for $Sys(\mathcal{G}, C)$ to have integral-action, it is assumed that the plant $\mathcal{G}$ does not have transmission-zeros or blocking-zeros at $s = 0$ whenever the design requirements include integral-action.

Fact 3: **a)** Let $\beta \in \mathbb{R}_+$. Define $\chi := \dfrac{s\,[\,(s+\beta)^q - s^q\,]}{(s+\beta)^q}$. For any integer $q \geq 0$, $\|\,\chi\,\| = q\,\beta$. **b)** Let $\alpha \in \mathbb{R}_+$. Define $\tilde{\chi} := \dfrac{(s+\alpha)^r - \alpha^r}{s\,(s+\alpha)^r}$. For any integer $r \geq 1$, $\|\,\tilde{\chi}\,\| = \dfrac{r}{\alpha}$. □

III. Plants with Unrestricted Zeros

This section considers systems that have all delay terms in the numerator of the plant's transfer-function (matrix) given by either an LCF or an RCF:

Case 1) Let $\mathcal{G} = Y^{-1}\mathcal{X}$ be the plant with all delay terms in the numerator $\mathcal{X} \in \mathcal{M}(\mathcal{H}_\infty)$, and $Y \in \mathcal{M}(\mathbf{S})$ is delay-free; $\mathcal{G} \in \mathcal{M}(\mathcal{H}_\infty)$ if and only if $Y^{-1} \in \mathcal{M}(\mathbf{S})$. Let $\mathcal{X}_{k\ell}$ denote the (k,ℓ) entry of $\mathcal{X}$; each $\mathcal{X}_{k\ell}$ may contain any known delay terms. It is assumed that $\mathcal{X} = [\,\mathcal{X}_{k\ell}\,]_{k,\ell\in\{1,\dots,m\}} = [\,e^{-sh_{k\ell}}X_{k\ell}\,]_{k,\ell\in\{1,\dots,m\}}$, where $h_{k\ell} \geq 0$ is time delay in seconds. Let $\mathcal{G}$ have full (normal) rank m, and have no zeros at $s = 0$, equivalently, rank$\mathcal{X}(0) = m$. Define $X_o := \mathcal{X}(0) = (Y(s)\mathcal{G}(s))|_{s=0}$; then $X_o^{-1} = (\mathcal{G}^{-1}(s)Y^{-1}(s))|_{s=0}$. It is assumed that Y^{-1} may have poles anywhere in $\mathbb{C}_-$, but the only $\mathcal{U}$-poles are all at $s = 0$; equivalently, rank$Y(0) \leq m$, but rank$Y(s) = m$ for all $s \in \mathcal{U} \setminus \{0\}$. If Y^{-1} has poles at $s = 0$, then $Y^{-1}(s) \notin \mathcal{M}(\mathbf{S})$. The entries of Y^{-1} may have different multiplicities of poles at $s = 0$; some entries may have only poles in $\mathbb{C}_-$. Let $Y_{k\ell}(s)$ denote the (k,ℓ) entry of $Y^{-1}(s) = [\,Y_{k\ell}(s)\,]_{k,\ell=1,\dots,m}$. Define $q_{k\ell} \geq 0$ as the number of poles of $Y_{k\ell}(s)$ at $s = 0$, and $q_\ell := \max\limits_{1\leq k\leq m} q_{k\ell}$ as the largest number of poles at $s = 0$ of the entries in the ℓ-th column of $Y^{-1}(s)$. If all entries in the ℓ-th column of Y^{-1} are stable, then $q_\ell = 0$. Although $Y_{k\ell} \notin \mathbf{S}$ when $q_{k\ell} \neq 0$, $(Y_{k\ell}(s)\frac{s^{q_\ell}}{(s+\beta)^{q_\ell}}) \in \mathbf{S}$ for any $\beta \in \mathbb{R}_+$.

Case 2) Let $\mathcal{G} = \widetilde{\mathcal{X}}\widetilde{Y}^{-1}$ be the plant, with the delay terms all in $\widetilde{\mathcal{X}} \in \mathcal{M}(\mathcal{H}_\infty)$; $\widetilde{Y} \in \mathcal{M}(\mathbf{S})$ is delay-free; $\mathcal{G} \in \mathcal{M}(\mathcal{H}_\infty)$ if and only if $\widetilde{Y}^{-1} \in \mathcal{M}(\mathbf{S})$. The entries $\widetilde{\mathcal{X}}_{k\ell}$, $\widetilde{Y}_{k\ell}$ are similar; $\widetilde{\mathcal{X}} = \left[\,\widetilde{\mathcal{X}}_{k\ell}\,\right]_{k,\ell\in\{1,\dots,m\}} = \left[\,e^{-sh_{k\ell}}\widetilde{X}_{k\ell}\,\right]_{k,\ell\in\{1,\dots,m\}}$, $\widetilde{Y}^{-1}(s) = \left[\widetilde{Y}_{k\ell}(s)\right]_{k,\ell=1,\dots,m}$. Define $\widetilde{X}_o := \widetilde{\mathcal{X}}(0) = (\mathcal{G}(s)\widetilde{Y}(s))|_{s=0}$; then $\widetilde{X}_o^{-1} = (\widetilde{Y}^{-1}(s)\mathcal{G}^{-1}(s))|_{s=0}$. The poles of $\widetilde{Y}^{-1}$ are similar; rank$\widetilde{Y}(0) \leq m$, but rank$\widetilde{Y}(s) = m$ for all $s \in \mathcal{U} \setminus \{0\}$. Define $q_{k\ell} \geq 0$ as the number of poles at $s = 0$ of the entries $\widetilde{Y}_{k\ell}(s)$ of $\widetilde{Y}^{-1}(s)$. Define $q_k := \max\limits_{1\leq\ell\leq m} q_{k\ell}$ as the largest number of poles at $s = 0$ of the entries in the k-th row of $\widetilde{Y}^{-1}(s)$. If all entries in the k-th row of $\widetilde{Y}^{-1}$ are stable, then $q_k = 0$. Although $\widetilde{Y}_{k\ell}(s) \notin \mathbf{S}$ when $q_{k\ell} \neq 0$, $(\frac{s^{q_k}}{(s+\beta)^{q_k}}\widetilde{Y}_{k\ell}(s)) \in \mathbf{S}$ for any $\beta \in \mathbb{R}_+$.

Theorem 1 presents finite-dimensional stabilizing controller synthesis for the two classes of MIMO plants. Theorem 1-(b) includes integral-action in the stabilizing controller.

Theorem 1: MIMO stabilizing controller synthesis:

Case 1) Let $\mathcal{G} = Y^{-1}\mathcal{X}$. **a)** Choose $\beta \in \mathbb{R}_+$ such that $\beta < (\max\limits_\ell q_\ell\,\|\,\frac{1}{s}\,[\,\mathcal{X}(s)X_o^{-1} - I\,]\,\|\,)^{-1}$. For $\ell = 1,\dots,m$, define $\psi_\ell(s) := [\,(s+\beta)^{q_\ell} - s^{q_\ell}\,]$. Then the controller C in (2) stabilizes $\mathcal{G}$:

$$C = X_o^{-1}\mathrm{diag}\left[\frac{\psi_1(s)}{s^{q_1}}\,,\quad\cdots\quad,\frac{\psi_m(s)}{s^{q_m}}\right]Y(s)\ . \tag{2}$$

b) Choose $\tilde{\beta} \in \mathbb{R}_+$ such that $\tilde{\beta} < ((1 + \max\limits_\ell q_\ell)\,\|\,\frac{1}{s}\,[\mathcal{X}(s)X_o^{-1} - I]\,\|)^{-1}$. For $\ell = 1,\dots,m$, define $\tilde{\psi}_\ell(s) := [\,(s+\tilde{\beta})^{1+q_\ell} - s^{1+q_\ell}\,]$. Then the integral-action controller C_I in (3) stabilizes $\mathcal{G}$:

$$C_I = \frac{1}{s}\,X_o^{-1}\mathrm{diag}\left[\frac{\tilde{\psi}_1(s)}{s^{q_1}}\,,\quad\cdots\quad,\frac{\tilde{\psi}_m(s)}{s^{q_m}}\right]Y(s)\ . \tag{3}$$

Case 2) Let $\mathcal{G} = \widetilde{\mathcal{X}}\widetilde{Y}^{-1}$. **a)** Choose $\beta \in \mathbb{R}_+$ such that $\beta < (\max\limits_k q_k\,\|\,\frac{1}{s}\,[\,\widetilde{X}_o^{-1}\widetilde{\mathcal{X}}(s) - I\,]\,\|\,)^{-1}$. For $k = 1,\dots,m$, define $\psi_k(s) := [\,(s+\beta)^{q_k} - s^{q_k}\,]$. Then the controller C in (4) stabilizes $\mathcal{G}$:

$$C = \widetilde{Y}(s)\,\mathrm{diag}\left[\frac{\psi_1(s)}{s^{q_1}}\,,\quad\cdots\quad,\frac{\psi_m(s)}{s^{q_m}}\right]\widetilde{X}_o^{-1}\ . \tag{4}$$

b) Choose $\tilde{\beta} \in \mathbb{R}_+$ such that $\tilde{\beta} < ((1+\max_k q_k)\,\|\frac{1}{s}[\,\widetilde{X}_o^{-1}\mathcal{X}(s)-I\,]\,\|)^{-1}$. For $k=1,\ldots,m$, define $\tilde{\psi}_k(s) := [\,(s+\tilde{\beta})^{1+q_k} - s^{1+q_k}\,]$. Then the integral-action controller C_I in (5) stabilizes $\mathcal{G}$:

$$C_I = \frac{1}{s}\,\widetilde{Y}(s)\,\mathrm{diag}\left[\frac{\tilde{\psi}_1(s)}{s^{q_1}}, \quad \ldots \quad, \frac{\tilde{\psi}_m(s)}{s^{q_m}}\right]\widetilde{X}_o^{-1}\ . \tag{5}$$

□

In Theorem 1 Case-(1a), if $q_\ell = 0$ for some $\ell \in \{1,\ldots,m\}$, then the corresponding $\psi_\ell = 0$. If $\mathcal{G} \in \mathcal{M}(\mathcal{H}_\infty)$, then $Y^{-1} \in \mathcal{M}(\mathbf{S})$ and hence, $\psi_\ell = 0$ for all $\ell \in \{1,\ldots,m\}$. Therefore, the controller $C = 0$ in (2). Theorem 1 Case-(1b) gives a non-zero integral-action controller $C_I = \frac{\tilde{\beta}}{s}X_o^{-1}Y(s)$. as in (3). A more general design for the case when $\mathcal{G} \in \mathcal{M}(\mathcal{H}_\infty)$ is proposed in Theorem 2. These are (realizable) PID controllers of the form

$$C_{pid}(s) = K_p + \frac{s}{\tau\,s+1}K_d + \frac{1}{s}K_i\ , \tag{6}$$

where $\tau \in \mathbb{R}_+$ (typically very small).

Theorem 2: Controller synthesis when $\mathcal{G}$ is stable:
Let $\mathcal{G} \in \mathcal{M}(\mathcal{H}_\infty)$. **a)** Choose any $\tau \in \mathbb{R}_+$, $K_p \in \mathbb{R}^{m\times m}$, $K_d \in \mathbb{R}^{m\times m}$, $Q \in \mathbf{S}^{m\times m}$, $K_i = 0$, and $\beta \in \mathbb{R}_+$ such that $\beta < \|\,\mathcal{G}\,(\,K_p+\frac{s}{\tau\,s+1}K_d\,)Q\,\|^{-1}$. Then the stable controller C in (7) stabilizes $\mathcal{G}$:

$$C = \beta\,(\,K_p + \frac{s}{\tau\,s+1}K_d\,)Q\ . \tag{7}$$

b) Choose any $\tau \in \mathbb{R}_+$, $K_p \in \mathbb{R}^{m\times m}$, $K_d \in \mathbb{R}^{m\times m}$. Let $K_i = \mathcal{G}(0)^{-1}$. Choose $Q \in \mathbf{S}^{m\times m}$ such that $Q(0) = I$, $\beta \in \mathbb{R}_+$ such that $\beta < \|\mathcal{G}(\,K_p+\frac{s}{\tau\,s+1}K_d+\frac{1}{s}K_i)Q-\frac{1}{s}\,I\,\|^{-1}$. Then the integral-action controller C_I in (8) stabilizes $\mathcal{G}$:

$$C_I = \beta\,(\,K_p + \frac{s}{\tau\,s+1}K_d + \frac{1}{s}\,K_i\,)Q\ . \tag{8}$$

□

If $Q \in \mathbf{S}^{m\times m}$ is chosen as $Q = I_m$, then the stable C in (7) becomes a PD controller, and the integral-action C_I in (8) becomes a PID controller. Examples of interconnections where Theorem 1 can be applied are given next.

A. Systems with input or output delays

Systems subject to input or output delays are the simplest systems containing delay terms in only the plant's numerator. Let $\mathcal{E} = \left[e^{-sh_{k\ell}}\right]_{k,\ell=1,\ldots,m} \in \mathcal{H}_\infty{}^{m\times m}$ denote a matrix of arbitrary delay terms, where $e^{-sh_{k\ell}}$ represents a delay of $h_{k\ell}$ seconds. Let $G \in \mathbf{R_p}^{m\times m}$ be delay-free, and have no zeros at $s=0$. Let G have poles anywhere in $\mathbb{C}_-$, but the only $\mathcal{U}$-poles are at $s=0$. In Fig. 2, $\mathcal{E}$ represents input delays in the system $\mathcal{G}_i = G\,\mathcal{E}$. With an LCF $G = Y^{-1}X$, the system $\mathcal{G}_i = Y^{-1}(X\mathcal{E})$ has all delay terms in $\mathcal{X} := (X\mathcal{E}) \in \mathcal{M}(\mathcal{H}_\infty)$, and $Y \in \mathcal{M}(\mathbf{S})$ is delay-free. Therefore, the controller design results of Theorem 1-Case (1) can be applied by substituting $\mathcal{X} = (X\mathcal{E})$. With $\mathcal{E}(0) = I_m$, we have $X_o = X(0) = (\,Y(s)\mathcal{G}(s)\,)|_{s=0}$. Similarly, $\mathcal{E}$ represents output delays in the system $\mathcal{G}_o = \mathcal{E}\,G$. With an RCF $G = \widetilde{X}\widetilde{Y}^{-1}$, the system $\mathcal{G}_o = (\mathcal{E}\widetilde{X})\widetilde{Y}^{-1}$ has all delay terms $\widetilde{\mathcal{X}} := (\mathcal{E}\widetilde{X}) \in \mathcal{M}(\mathcal{H}_\infty)$, and $\widetilde{Y} \in \mathcal{M}(\mathbf{S})$ is delay-free. Therefore, the controller design results of Theorem 1-Case (2) can be applied by substituting $\widetilde{\mathcal{X}} = (\mathcal{E}\widetilde{X})$. With $\mathcal{E}(0) = I_m$, we have $\widetilde{X}_o = \widetilde{X}(0) = (\,\mathcal{G}(s)\widetilde{Y}(s)\,)|_{s=0}$.

B. Interconnected systems with cascade delay

The interconnected system $\mathcal{G}_c$ in Fig. 3 has delay-free subsystems $P, G \in \mathbf{R_p}^{m\times m}$. Let $\mathcal{E} = \left[\,e^{-sh_{k\ell}}\,\right]_{k,\ell=1,\ldots,m} \in \mathcal{M}(\mathcal{H}_\infty)$ denote arbitrary delay terms. Define $\mathcal{G} := P\mathcal{E}G$. The $(m\times 2m)$ transfer-function from (u,v) to y is $\mathcal{G}_c = (I+P\mathcal{E}G)^{-1}\left[\,P\mathcal{E}G \quad P\,\right] = (I+\mathcal{G})^{-1}\left[\,\mathcal{G} \quad P\,\right]$. In $\mathcal{G}_c$, the delay-free P and G may have poles anywhere in $\mathbb{C}_-$, but the only $\mathcal{U}$-poles, if any, are all at $s=0$; they have no $\mathcal{U}$-zeros at $s=0$.

Case 1) Suppose for some LCF $G = D_g^{-1}N_g$, $P = D_p^{-1}N_p$ that *i)* $D_g = \frac{s^\mu}{(s+a)^\mu}I_m$, $\mu \geq 0$ is an integer, $a \in \mathbb{R}_+$, or that *ii)* N_p, D_g, $\mathcal{E}$ are all diagonal. Condition (i) on D_g also includes stable G since $D_g = I$ for $\mu = 0$ implies $G \in \mathcal{M}(\mathbf{S})$. The $\mathcal{U}$-poles of D_p^{-1}, D_g^{-1}, if any, are at $s=0$. Furthermore, rank$N_p(0) = m$, rank$N_g(0) = m$ since P, G have no zeros at $s=0$. Both conditions imply D_g commutes with $N_p\mathcal{E}$, and $\mathcal{G} = P\mathcal{E}G = (D_gD_p)^{-1}(N_p\mathcal{E}N_g) = Y^{-1}\mathcal{X}$. Then $\mathcal{G} = Y^{-1}\mathcal{X}$ has all delay terms in the numerator $\mathcal{X} := (N_p\,\mathcal{E}N_g) \in \mathcal{M}(\mathcal{H}_\infty)$, and the $\mathcal{U}$-poles of the entries of $Y^{-1} = (D_gD_p)^{-1}$ are all at $s=0$. The interconnected system $\mathcal{G}_c$ in Fig. 3 is then described as $\mathcal{G}_c = (I+\mathcal{G})^{-1}\left[\,\mathcal{G} \quad P\,\right] = (Y+\mathcal{X})^{-1}\left[\,\mathcal{X} \quad D_gN_p\,\right] = (D_gD_p+N_p\,\mathcal{E}N_g)^{-1}N_p\left[\,\mathcal{E}N_g \quad D_g\,\right]$.

Case 2) Let $P = D_p^{-1}N_p$ be an LCF, where rank$N_p(0) = m$ since P has no transmission-zeros at $s=0$. Suppose that $D_p = \frac{s^\rho}{(s+a)^\rho}I_m$, where $\rho \geq 0$ is an integer, and $a \in \mathbb{R}_+$. This condition also includes the case of stable P since $D_p = I$ for $\rho = 0$ implies $P \in \mathcal{M}(\mathbf{S})$. Let $G = \widetilde{N}_g\widetilde{D}_g^{-1}$ be an RCF, where rank$\widetilde{N}_g(0) = m$ since G has no transmission-zeros at $s=0$. Under these assumptions, $\mathcal{G} = P\mathcal{E}G = (\,N_p\,\mathcal{E}\widetilde{N}_g\,)(D_p\widetilde{D}_g)^{-1} = \widetilde{\mathcal{X}}\widetilde{Y}^{-1}$. Then $\mathcal{G} = \widetilde{\mathcal{X}}\widetilde{Y}^{-1}$ has all delay terms in $\widetilde{\mathcal{X}} = N_p\,\mathcal{E}\widetilde{N}_g \in \mathcal{M}(\mathcal{H}_\infty)$, and the $\mathcal{U}$-poles of $\widetilde{Y}^{-1} = \widetilde{D}_g^{-1}D_p^{-1}$ are all at $s=0$. The interconnected system $\mathcal{G}_c$ in Fig. 3 is then described as $\mathcal{G}_c = (I+\mathcal{G})^{-1}\left[\,\mathcal{G} \quad P\,\right] = \left[\widetilde{\mathcal{X}}(\widetilde{Y}+\widetilde{\mathcal{X}})^{-1} \quad \widetilde{D}_g(\widetilde{Y}+\widetilde{\mathcal{X}})^{-1}N_p\right]$.

Proposition 1 designs controllers that stabilize the interconnected system $\mathcal{G}_c$ as shown in the feedback configuration $Sys(\mathcal{G}_c, C_g)$ of Fig. 3 based on the two cases of Theorem 1.

Proposition 1: (Design for interconnections with cascade delay): Consider the system $Sys(\mathcal{G}_c, C_g)$.

Case 1) Let $\mathcal{G}$ satisfy the assumptions of Case (1). Let C stabilizing $\mathcal{G}$ be as in (2), and let C_I stabilizing $\mathcal{G}$ be as in (3). Then $C_g = (\,C - I\,)$ is a controller that stabilizes $\mathcal{G}_c$, and $C_{Ig} = (\,C_I - I\,)$ is an integral-action controller that stabilizes $\mathcal{G}_c$ in the feedback configuration $Sys(\mathcal{G}_c, C_g)$.

Case 2) Let $\mathcal{G}$ satisfy the assumptions of Case (2). Let C stabilizing $\mathcal{G}$ be as in (4), and let C_I stabilizing $\mathcal{G}$ be as in (5). Then $C_g = (\,C - I\,)$ is a controller that stabilizes

$\mathcal{G}_c$, and $C_{Ig} = (\,C_I - I\,)$ is an integral-action controller that stabilizes $\mathcal{G}_c$ in the feedback configuration $Sys(\mathcal{G}_c, C_g)$. □

IV. MIMO PLANTS WITH UNRESTRICTED POLES

This section considers systems where the delay terms are all in the denominator of the plant's transfer-function (matrix). As in Section III, there are two cases that describe such plants given by either an LCF or an RCF:

Case 1) Let $\mathcal{G} = \mathcal{Y}^{-1}X$ be the plant, with all delay terms in $\mathcal{Y} \in \mathcal{M}(\mathcal{H}_\infty)$, and $X \in \mathcal{M}(\mathbf{S})$ is delay-free. Let $Y_n(s) \in \mathcal{M}(\mathbf{S})$, $\det Y_n(\infty) \neq 0$, $\mathcal{Y}_d = \sum_{i=1}^{\nu} e^{-sh_i} Q_i(s)$, $Q_i(\infty) = 0$, $h_i \geq 0$, $i = 1, \ldots, \nu$. Define $Y_n(\infty) := \mathcal{Y}(\infty) = (X(s)\mathcal{G}^{-1}(s))|_{s\to\infty}$; $Y_n(\infty)^{-1} = (\,\mathcal{G}(s)X^{-1}(s)\,)|_{s\to\infty}$. The zeros of $\mathcal{G}$ are all in $\mathbb{C}_-$, and at infinity; equivalently, $\operatorname{rank} X(\infty) \leq m$, but $\operatorname{rank} X(s) = m$ for all $s \in \mathcal{U} \setminus \{\infty\}$. Therefore, X^{-1} has no poles in $s \in \mathcal{U} \setminus \{\infty\}$, but may have poles at infinity, i.e., if $\operatorname{rank} X(\infty) < m$, then $X^{-1} \notin \mathcal{M}(\mathbf{S})$ because it is improper. With $n_{k\ell}$, $d_{k\ell}$ as numerator and denominator polynomials of the (k,ℓ) entry, write $X^{-1}(s) = \left[\dfrac{n_{k\ell}(s)}{d_{k\ell}(s)}\right]_{k,\ell\in\{1,\ldots,m\}}$. Define the integers $r_{k\ell} := \begin{cases} \delta(n_{k\ell}) - \delta(d_{k\ell}), & \text{if } \delta(n_{k\ell}) > \delta(d_{k\ell}) \\ 0\,, & \text{if } \delta(n_{k\ell}) \leq \delta(d_{k\ell}) \end{cases}$, $r_\ell := \max_{1\leq k\leq m} r_{k\ell}$, $\ell = 1, \ldots, m$. Let $\xi_\ell(s)$ be any monic r_ℓ-th order strictly Hurwitz polynomial, $\ell = 1, \ldots, m$; e.g., $\xi_\ell(s) = (s+a)^{r_\ell}$ for $a \in \mathbb{R}_+$. If $r_\ell = 0$, then $\xi_\ell = 1$. Define $\Delta(s) := \operatorname{diag}\left[\xi_1(s), \ \cdots \ , \xi_m(s)\right]$. Although $X^{-1} \notin \mathcal{M}(\mathbf{S})$ when $\operatorname{rank} X(\infty) < m$, the order of ξ_ℓ is chosen to make each entry $\frac{n_{k\ell}(s)}{d_{k\ell}(s)\xi_\ell(s)}$ proper; hence, $X^{-1}\Delta^{-1}$ is stable since the polynomials ξ_ℓ and $d_{k\ell}$ are strictly Hurwitz.

Case 2) Let the delayed plant be $\mathcal{G} = \widetilde{X}\widetilde{\mathcal{Y}}^{-1}$; the delay terms are all in $\widetilde{\mathcal{Y}} \in \mathcal{M}(\mathcal{H}_\infty)$, and $\widetilde{X} \in \mathcal{M}(\mathbf{S})$ is delay-free. Let $\widetilde{\mathcal{Y}} = \widetilde{Y}_n + \widetilde{\mathcal{Y}}_d$, $\widetilde{Y}_n(s) \in \mathcal{M}(\mathbf{S})$, $\det \widetilde{Y}_n(\infty) \neq 0$, $\widetilde{\mathcal{Y}}_d = \sum_{i=1}^{\nu} e^{-sh_i}\widetilde{Q}_i(s)$, $\widetilde{Q}_i(\infty) = 0$. Define $\widetilde{Y}_n(\infty) := \widetilde{\mathcal{Y}}(\infty) = (\widetilde{X}(s)\mathcal{G}^{-1}(s))|_{s\to\infty}$; $\widetilde{Y}_n(\infty)^{-1} = (\,\mathcal{G}(s)\widetilde{X}^{-1}(s)\,)|_{s\to\infty}$. The zeros of $\mathcal{G}$ are similar; $\operatorname{rank}\widetilde{X}(\infty) \leq m$, but $\operatorname{rank}\widetilde{X}(s) = m$ for all $s \in \mathcal{U} \setminus \{\infty\}$. Therefore, $\widetilde{X}^{-1}$ has no poles in $s \in \mathcal{U} \setminus \{\infty\}$, but may have poles at infinity, i.e., if $\operatorname{rank}\widetilde{X}(\infty) < m$, then $\widetilde{X}^{-1} \notin \mathcal{M}(\mathbf{S})$ because it is improper. With $\tilde{n}_{k\ell}$, $\tilde{d}_{k\ell}$ as numerator and denominator polynomials of the (k,ℓ) entry, write $\widetilde{X}^{-1}(s) = \left[\dfrac{\tilde{n}_{k\ell}(s)}{\tilde{d}_{k\ell}(s)}\right]_{k,\ell\in\{1,\ldots,m\}}$. Define the integers $r_{k\ell} := \begin{cases} \delta(\tilde{n}_{k\ell}) - \delta(\tilde{d}_{k\ell})\,, & \text{if } \delta(\tilde{n}_{k\ell}) > \delta(\tilde{d}_{k\ell}) \\ 0\,, & \text{if } \delta(\tilde{n}_{k\ell}) \leq \delta(\tilde{d}_{k\ell}) \end{cases}$, $r_k := \max_{1\leq \ell\leq m} r_{k\ell}$, $k = 1, \ldots, m$. Let $\xi_k(s)$ be any monic r_k-th order strictly Hurwitz polynomial, $k = 1, \ldots, m$; e.g., $\xi_k(s) = (s+a)^{r_k}$ for $a \in \mathbb{R}_+$. If $r_k = 0$, then $\xi_k = 1$. Define $\Delta(s) := \operatorname{diag}\left[\xi_1(s), \ \cdots \ , \xi_m(s)\right]$. Although $\widetilde{X}^{-1} \notin \mathcal{M}(\mathbf{S})$ when $\operatorname{rank}\widetilde{X}(\infty) < m$, the order of ξ_k is chosen to make each entry $\frac{\tilde{n}_{k\ell}(s)}{\tilde{d}_{k\ell}(s)\xi_\ell(s)}$ proper; hence, $\Delta^{-1}\widetilde{X}^{-1}$ is stable since ξ_k and $\tilde{d}_{k\ell}$ are strictly Hurwitz.

For the two classes of plants described as $\mathcal{G} = \mathcal{Y}^{-1}X$ of Case (1), and $\mathcal{G} = \widetilde{X}\widetilde{\mathcal{Y}}^{-1}$ of Case (2), Theorem 3 presents finite-dimensional controller synthesis methods for closed-loop stability. These controllers all have integral-action.

Theorem 3: MIMO stabilizing controller synthesis:

Case 1) Let $\mathcal{G} = \mathcal{Y}^{-1}X = (Y_n + \mathcal{Y}_d)^{-1}X$. Define $\rho_\ell := \begin{cases} 1\,, & \text{if } r_\ell = 0 \\ r_\ell\,, & \text{if } r_\ell \geq 1\,. \end{cases}$, $\ell = 1, \ldots, m$. Let $\alpha \in \mathbb{R}_+$ be such that $\alpha > \max_\ell \rho_\ell \,\|\, s\,[\,\mathcal{Y}(s)Y_n(\infty)^{-1} - I\,]\,\|$. Define $\varphi_\ell(s) := [\,(s+\alpha)^{\rho_\ell} - \alpha^{\rho_\ell}\,]$, $\ell = 1, \ldots, m$. Then the controller C in (9) stabilizes $\mathcal{G}$:

$$C = X^{-1}(s)\operatorname{diag}\left[\frac{\alpha^{\rho_1}}{\varphi_1(s)}\,, \quad \cdots \quad , \frac{\alpha^{\rho_m}}{\varphi_m(s)}\right] Y_n(\infty)\,. \quad (9)$$

Case 2) Let $\mathcal{G} = \widetilde{X}\widetilde{\mathcal{Y}}^{-1} = \widetilde{X}(\widetilde{Y}_n + \widetilde{\mathcal{Y}}_d)^{-1}$. Define $\rho_k := \begin{cases} 1\,, & \text{if } r_k = 0 \\ r_k\,, & \text{if } r_k \geq 1\,. \end{cases}$, $k = 1, \ldots, m$. Let $\alpha \in \mathbb{R}_+$ be such that $\alpha > \max_k \rho_k \,\|\, s\,[\,\widetilde{Y}_n(\infty)^{-1}\widetilde{\mathcal{Y}}(s) - I\,]\,\|$. Define $\varphi_k(s) := [\,(s+\alpha)^{\rho_k} - \alpha^{\rho_k}\,]$, $k = 1, \ldots, m$. Then the controller C in (10) stabilizes $\mathcal{G}$:

$$C = \widetilde{Y}_n(\infty)\operatorname{diag}\left[\frac{\alpha^{\rho_1}}{\varphi_1(s)}\,, \quad \cdots\,, \quad \frac{\alpha^{\rho_m}}{\varphi_m(s)}\right]\widetilde{X}^{-1}(s)\,. \quad (10)$$

□

In Theorem 3, C in (9) (and C in (10) has integral-action since the diagonal terms all have poles at $s = 0$. The terms corresponding to $r_\ell = 0$ are $\dfrac{\alpha}{s}$; those corresponding to $r_\ell \geq 1$ are $(\dfrac{\alpha^{r_\ell}}{(s+\alpha)^{r_\ell} - \alpha^{r_\ell}})$, with one pole at $s = 0$, and the remaining $(r_\ell - 1)$ poles all in $\mathbb{C}_-$.

A. Interconnected systems with feedback delay

The interconnected system $\mathcal{G}_f$ in Fig. 4 has delay-free subsystems $P, G \in \mathbf{R_p}^{m\times m}$. Let $\mathcal{E} = \left[\,e^{-sh_{k\ell}}\,\right]_{k,\ell=1,\ldots,m} \in \mathcal{M}(\mathcal{H}_\infty)$ be a matrix of delay terms. Define $\mathcal{G} := (I + PG\mathcal{E})^{-1}PG$; then $\mathcal{G}_f = (I + PG\mathcal{E})^{-1}[\,PG \quad P\,] = \left[\mathcal{G} \quad (I - \mathcal{G}\mathcal{E})P\right]$ is the transfer-function from (u, v) to y. The delay-free subsystems P and G have unrestricted poles anywhere in $\mathbb{C}$. The only transmission-zeros and blocking-zeros of P and G are all in $\mathbb{C}_-$, and at infinity.

Case 1) Suppose that any of the following three conditions hold for some LCF $G = D_g^{-1}N_g$, and $P = D_p^{-1}N_p$: *i)* $N_p = \dfrac{1}{(s+a)^\varrho}I_m$, $\varrho \geq 0$ is an integer, $a \in \mathbb{R}_+$, or *ii)* $D_g = d_g I_m$, $d_g \in \mathbf{S}$, or *iii)* N_p and D_g are diagonal. Let $(N_pN_g)(\infty) = 0$. If N_p, D_g satisfy any of the three given conditions, then they commute, $N_pD_g^{-1} = D_g^{-1}N_p$. Under these assumptions, $\mathcal{G} = (I + PG\mathcal{E})^{-1}PG = (D_gD_p + N_pN_g\mathcal{E})^{-1}(N_pN_g) = \mathcal{Y}^{-1}X$ has all delay terms in $\mathcal{Y} = (D_gD_p + N_pN_g\, l\mathcal{E}) \in \mathcal{M}(\mathcal{H}_\infty)$, and the $\mathcal{U}$-zeros of $X = (N_pN_g)$ are all at infinity. Furthermore, $\mathcal{Y} = (Y_n + \mathcal{Y}_d)$; $Y_n = (D_gD_p)$ is delay-free, and $\mathcal{Y}_d(\infty) = (N_pN_g\,\mathcal{E})(\infty) = 0$ since $(N_pN_g)(\infty) = 0$. Therefore, $\mathcal{G} = \mathcal{Y}^{-1}X$. The system $\mathcal{G}_f$ in Fig. 4 is described as $\mathcal{G}_f = (I + PG\mathcal{E})^{-1}[PG \quad P] = \mathcal{Y}^{-1}[X \quad D_gN_p] = (D_gD_p + N_pN_g\,\mathcal{E})^{-1}N_p\,[N_g \quad D_g]$.

Case 2) Let P be stable. Let $G = \widetilde{N}_g\widetilde{D}_g^{-1}$ be an RCF of G. Let $(P\widetilde{N}_g)(\infty) = 0$. Under these assumptions, $\mathcal{G} = PG(I+\mathcal{E}PG)^{-1} = P\widetilde{N}_g\widetilde{D}_g^{-1}(I+\mathcal{E}P\widetilde{N}_g\widetilde{D}_g^{-1})^{-1} = (P\widetilde{N}_g)(\widetilde{D}_g+\mathcal{E}P\widetilde{N}_g)^{-1} = \widetilde{X}\widetilde{\mathcal{Y}}^{-1}$ has all delay terms in $\widetilde{\mathcal{Y}} = (\widetilde{D}_g+\mathcal{E}P\widetilde{N}_g) \in \mathcal{M}(\mathcal{H}_\infty)$, and the $\mathcal{U}$-zeros of $\widetilde{X} = (P\widetilde{N}_g)$ are all at infinity. Furthermore, $\widetilde{\mathcal{Y}} = (\widetilde{Y}_n + \widetilde{\mathcal{Y}}_d)$; $\widetilde{Y}_n = \widetilde{D}_g$ is delay-free, and $\mathcal{Y}_d(\infty) = (\mathcal{E}P\widetilde{N}_g)(\infty) = 0$ since $(P\widetilde{N}_g)(\infty) = 0$. Therefore, $\mathcal{G} = \widetilde{X}\widetilde{\mathcal{Y}}^{-1}$ is in the form of Theorem 3-Case 2. The system $\mathcal{G}_f$ in Fig. 4 is described as $\mathcal{G}_f = \begin{bmatrix}\mathcal{G} & (I-\mathcal{G}\mathcal{E})P\end{bmatrix} = \begin{bmatrix}\widetilde{X} & P\end{bmatrix}\begin{bmatrix}\widetilde{\mathcal{Y}} & \mathcal{E}P \\ 0 & I\end{bmatrix}^{-1}$.

Based on the two cases of Theorem 3, Proposition 2 designs controllers stabilizing the interconnected system $\mathcal{G}_f$ as shown in the feedback configuration $Sys(\mathcal{G}_f, C)$ of Fig. 4.

Proposition 2: (Design for interconnections with feedback delay): Consider the system $Sys(\mathcal{G}_f, C)$.

Case 1) Let $\mathcal{G}$ be satisfy the assumptions of Case (1). A controller C as in (9) that stabilizes $\mathcal{G}$ also stabilizes $\mathcal{G}_f$ in the feedback configuration $Sys(\mathcal{G}_f, C)$.

Case 2) Let $\mathcal{G}$ satisfy the assumptions of Case (2). A controller C as in (10) that stabilizes $\mathcal{G}$ also stabilizes $\mathcal{G}_f$ in the feedback configuration $Sys(\mathcal{G}_f, C)$. □

V. Load frequency control with delays

The goal of power system control is to maintain stability, performance and system integrity after failures occur, or when system disturbances such as short circuits and loss of generation are present. Time delays arising during transmission become important for maintaining stability. The controllers developed in the previous sections can be applied to single-area and complex multi-area interconnections of power systems. The designs can be tested using typical values of the model parameters given in e.g., [6], [10], [12].

Single-area load-frequency control with delays: Consider the load-frequency control problem of one generator supplying power to a single service area as in Fig. 5: A linearized low-order model of the plant for purposes of system frequency analysis and control synthesis consists of three main parts. The transfer-functions of the load and machine, speed governor, and turbine of the j-the service area are $F_{pj}(s)$, $F_{gj}(s)$, $F_{tj}(s)$. The speed regulation due to governor action is represented by the constant speed droop characteristic R_j. Let T_{pj}, T_{gj}, T_{tj}, T_{rj} be the time-constants of the load, governor, non-reheated turbine, and reheated turbine; κ_{pj} is a constant inversely proportional to the generator damping coefficient, c_{rj} is a constant for reheated turbine. For all types of turbines, the load transfer-function is $F_{pj}(s) = \frac{\kappa_{pj}}{T_{pj}s+1}$. For non-reheated and reheated turbines, the governor transfer-function is $F_{gj}(s) = \frac{1}{T_{gj}s+1}$; for hydraulic turbines, $F_{gj}(s) = \frac{1}{(T_{gj}s+1)} \cdot \frac{(T_{cj}s+1)}{(\frac{R_{tj}}{R_j}T_{cj}s+1)}$. The governors of hydraulic units include transient droop compensation for stable speed control performance [6]. The turbine transfer-function is $F_{tj}(s) = \frac{1}{T_{tj}s+1}$ for non-reheated turbines, $F_{tj}(s) = \frac{c_{rj}T_{rj}s+1}{(T_{rj}s+1)(T_{tj}s+1)}$ for reheated turbines, and $F_{tj}(s) = \frac{1-T_{wj}s}{0.5T_{wj}s+1}$ for hydraulic turbines, with $F_{tj}(s)$ containing a zero in $\mathbb{C}_+$ at $z_o = 1/T_{wj} \in \mathcal{U}$. Let e^{-sh_j} represent a delay of h_j seconds. Define $X_j := F_{pj}F_{tj}F_{gj}$, $\mathcal{X}_j := e^{-sh_j}X_j$, $Y_j := (I+R_j^{-1}X_j)$. For all three types of turbines used in generation, assuming that governor transfer-functions for hydraulic units include transient droop compensation as needed, R_j is such that $Y_j^{-1} \in \mathbf{S}$. The system in Fig. 5 has transfer-function $\mathcal{F}_j = (I+R_j^{-1}X_j)^{-1}\begin{bmatrix}(e^{-sh_j}X_j) & -F_{pj}\end{bmatrix} = Y_j^{-1}\begin{bmatrix}\mathcal{X}_j & -F_{pj}\end{bmatrix}\mathcal{M}(\mathcal{H}_\infty)$. Define $\mathcal{G}_j := (1+R_j^{-1}X_j)^{-1}(e^{-sh_j}X_j) = Y_j^{-1}\mathcal{X}_j \in \mathcal{H}_\infty$. Since the delay terms are all in the numerator $\mathcal{X} = (e^{-sh_j}X_j)$, the system $\mathcal{G}_j \in \mathcal{H}_\infty$ is as in Section III. With an RCF $C_j = \widetilde{N}_j\widetilde{D}_j^{-1}$, and $M_j = (Y_j\widetilde{D}_j + \mathcal{X}_j\widetilde{N}_j)$, $Sys(\mathcal{F}_j, C_j)$ is described as $\begin{bmatrix}x_j \\ f_j\end{bmatrix} = \begin{bmatrix}\widetilde{N}_j \\ -\widetilde{D}_j\end{bmatrix} M_j^{-1}\begin{bmatrix}Y_j & -\mathcal{X}_j & F_{pj}\end{bmatrix}\begin{bmatrix}r_j \\ w_j \\ v_j\end{bmatrix} + \begin{bmatrix}0 \\ r_j\end{bmatrix}$. The system $Sys(\mathcal{F}_j, C_j)$ is stable if and only if $M_j^{-1} \in \mathcal{H}_\infty$, equivalently, $C_j = \widetilde{N}_j\widetilde{D}_j^{-1}$ stabilizes $\mathcal{G}_j \in \mathcal{H}_\infty$, and C_j can be designed following Theorem 2. Choose any $\tau_j \in \mathbb{R}_+$, $K_{pj} \in \mathbb{R}$, $K_{dj} \in \mathbb{R}$, $Q_j \in \mathbf{S}$. For $\beta_j \in \mathbb{R}_+$ satisfying $\beta_j < \|\mathcal{G}_j(K_{pj} + \frac{s}{\tau_j s+1}K_{dj})Q_j\|^{-1}$, the stable $C_j = \beta_j(K_{pj} + \frac{s}{\tau_j s+1}K_{dj})Q_j \in \mathbf{S}$ stabilizes $\mathcal{G}_j$. Since $e^{-sh_j}(0) = 1$, $X_j(0) = \mathcal{X}_j(0) = F_{pj}(0) = \kappa_{pj}$, and $\mathcal{G}_j(0) = (1+R_j^{-1}\kappa_{pj})^{-1}\kappa_{pj}$. Let $K_{ij} = \mathcal{G}_j(0)^{-1} = (\kappa_{pj}^{-1} + R_j^{-1})$. Choose any $Q_j \in \mathbf{S}$ such that $Q_j(0) = 1$. For $\beta_j \in \mathbb{R}_+$ satisfying $\beta_j < \|\mathcal{G}_j(K_{pj} + \frac{s}{\tau_j s+1}K_{dj} + \frac{1}{s}(\kappa_j^{-1} + R_j^{-1}))Q_j - \frac{1}{s}I\|^{-1}$, the integral-action $C_{Ij} = \beta_j(K_{pj} + \frac{s}{\tau_j s+1}K_{dj} + \frac{1}{s}(\kappa_j^{-1} + R_j^{-1}))Q_j$ stabilizes $\mathcal{G}_j$. For low-order controllers, $Q_j \in \mathbf{S}$ is chosen as a constant, $Q_j = 1$; then the stable C_j becomes a PD, and the integral-action C_{Ij} becomes a PID controller.

Mul ti-area load-frequency control with delays: Consider the load-frequency control problem for m generators supplying power to m service areas as in Fig.6. A linearized low-order model is used for each of the service areas $j = 1, \dots, m$, which may have any of the three types of (non-reheated, reheated, hydraulic) turbines. In interconnected power systems, service areas are connected via tie-lines through which a power exchange occurs when the frequencies in connected areas are different. The tie-line power flows among the m areas are represented by $\mathcal{V}_T \in \mathcal{H}_\infty{}^{m\times m}$, which may be subject to delays. These tie-line delays may cause the entries of $\mathcal{V}_T$ to contain any arbitrary delay terms; $\mathcal{V}_T(0) \in \mathbb{R}^{m\times m}$ is a constant matrix, which may or may not be nonsingular. The $m\times m$ diagonal transfer-functions for the m-area system are $R = \text{diag}[R_1, \cdots, R_m]$, $F_p = \text{diag}[F_{p1}, \cdots, F_{pm}]$, $F_t = \text{diag}[F_{t1}, \cdots, F_{tm}]$, $F_g = \text{diag}[F_{g1}, \cdots, F_{gm}]$, $B = \text{diag}[B_1, \cdots, B_m]$; the non-zero constants $B_j \in \mathbb{R}$ are frequency bias factors for each of the m areas. They satisfy $(B+\mathcal{V}_T(0))$ is nonsingular. The system may be subject to additional communication delays in each of the m service areas, represented by $\mathcal{E} = \text{diag}\begin{bmatrix}e^{-sh_1}, \cdots, e^{-sh_m}\end{bmatrix}$. Define the diagonal matrices $X := F_pF_tF_g \in \mathbf{S}^{m\times m}$, $\mathcal{X} := X\mathcal{E} \in \mathcal{H}_\infty{}^{m\times m}$, $Y := (I+R^{-1}X) \in \mathbf{S}^{m\times m}$,

$\mathcal{G} := Y^{-1}X\mathcal{E} = Y^{-1}\mathcal{X} \in \mathcal{H}_\infty{}^{m\times m}$. For all three types of turbines used in generation, $Y^{-1} \in \mathcal{M}(\mathbf{S})$ assuming that governor transfer-functions for hydraulic units include transient droop compensation as needed. Therefore, $\mathcal{G} = Y^{-1}\mathcal{X} \in \mathcal{M}(\mathcal{H}_\infty)$. Let $\mathcal{Z} := Y + F_p\Phi\mathcal{V}_T = I + R^{-1}X + F_p\Phi\mathcal{V}_T$. Choose $\Phi \in \mathbb{R}^{m\times m}$ such that $\mathcal{Z}^{-1} \in \mathcal{M}(\mathcal{H}_\infty)$. There are many choices for $\Phi \in \mathbb{R}^{m\times m}$ that make $\mathcal{Z}$ unimodular. For example, since $Y^{-1} \in \mathcal{M}(\mathbf{S})$, and $\det(I + F_p\Phi\mathcal{V}_TY^{-1}) = \det(I + \Phi\mathcal{V}_TY^{-1}F_p)$, $\mathcal{Z}^{-1} \in \mathcal{M}(\mathcal{H}_\infty)$ for any $\Phi \in \mathbb{R}^{m\times m}$ such that $\|\Phi\| < \|\mathcal{V}_TY^{-1}F_p\|$. The transfer-function from (u,v) to y is $\mathcal{F}_T = (B+\mathcal{V}_T)(Y + F_p\Phi\mathcal{V}_T)^{-1}\left[X\mathcal{E} \quad -F_p\Phi\right] = (B+\mathcal{V}_T)\mathcal{Z}^{-1}\left[\mathcal{X} \quad -F_p\Phi\right]$. Define $\mathcal{G}_T = (B+\mathcal{V}_T)\,(Y + F_p\Phi\mathcal{V}_T)^{-1}X\mathcal{E} = (B+\mathcal{V}_T)\mathcal{Z}^{-1}\mathcal{X} \in \mathcal{M}(\mathcal{H}_\infty)$. Since $\mathcal{Z}^{-1}$ is stable, $\mathcal{G}_T \in \mathcal{M}(\mathcal{H}_\infty)$ satisfies the assumptions in Section III. Then $\mathcal{F}_T = \left[\mathcal{G}_T \quad -(B+\mathcal{V}_T)\mathcal{Z}^{-1}F_p\Phi\right]$. With $M = (\widetilde{D} + \mathcal{G}_T\widetilde{N})$, the system $Sys(\mathcal{F}_T, C)$ is described as $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \widetilde{N} \\ -\widetilde{D} \end{bmatrix} M^{-1} \left[I \quad -\mathcal{G}_T \quad (B+\mathcal{V}_T)\mathcal{Z}^{-1}F_p\Phi\right] \begin{bmatrix} r \\ w \\ v \end{bmatrix} + \begin{bmatrix} 0 \\ r \end{bmatrix}$. $Sys(\mathcal{F}_T, C)$ is stable if and only if $M^{-1} \in \mathcal{M}(\mathcal{H}_\infty)$, equivalently, C stabilizes $\mathcal{G}_T \in \mathcal{M}(\mathcal{H}_\infty)$, and C can be designed following Theorem 2. Choose any $\tau \in \mathbb{R}_+$, $K_p \in \mathbb{R}^{m\times m}$, $K_d \in \mathbb{R}^{m\times m}$, $Q \in \mathbf{S}^{m\times m}$. For $\beta \in \mathbb{R}_+$ satisfying $\beta < \|\mathcal{G}_T(K_p + \frac{s}{\tau s+1}K_d)Q\|^{-1}$, the stable $C = \beta(K_p + \frac{s}{\tau s+1}K_d)Q \in \mathbf{S}^{m\times m}$ stabilizes $\mathcal{G}_T$. Since $\mathcal{E}(0) = I_m$, we have $X(0) = \mathcal{X}(0) = F_p(0) = \mathrm{diag}[\,\kappa_{p1}, \cdots, \kappa_{pm}\,] =: \kappa$, and $\mathcal{G}_T(0) = (B + \mathcal{V}_T(0))\mathcal{Z}(0)^{-1}\mathcal{X}(0) = (B + \mathcal{V}_T(0))(I + R^{-1}\kappa + \kappa\Phi\mathcal{V}_T)^{-1}\kappa$. Let $K_i = \mathcal{G}_T(0)^{-1} = (\kappa^{-1} + R^{-1} + \Phi\mathcal{V}_T)(B + \mathcal{V}_T(0))^{-1}$. Choose any $Q \in \mathbf{S}^{m\times m}$ such that $Q(0) = I$. For $\beta \in \mathbb{R}_+$ satisfying $\beta < \|\mathcal{G}_T(K_p + \frac{s}{\tau s+1}K_d + \frac{1}{s}K_i)Q - \frac{1}{s}I\|^{-1}$, the integral-action controller $C_I = \beta[K_p + \frac{s}{\tau s+1}K_d + \frac{1}{s}(\kappa^{-1} + R^{-1} + \Phi\mathcal{V}_T)(B + \mathcal{V}_T(0))^{-1}]Q$ stabilizes $\mathcal{G}_T$. The finite-dimensional controllers here contain design parameter choices, which can be used to satisfy performance objectives.

REFERENCES

[1] R.F. Curtain, and H. Zwart, *An Introduction to Infinite-dimensional Linear Systems Theory*, Texts in Applied Mathematics, Vol. 21, Springer, New York, 1995.

[2] K. Gu, V. L. Kharitonov, and J. Chen, *Stability of Time-Delay Systems*, Birkhäuser, Boston, 2003.

[3] A. N. Gündeş, H. Özbay, and A. B. Özgüler, "PID controller synthesis for a class of unstable MIMO plants with I/O Delays," *Automatica*, vol. 43, no. 1, pp. 135-142, 2007.

[4] A. N. Gündeş, L. Chow, "Controller synthesis for single-area and multi-area power systems with communication delays," *American Control Conference*, pp. 970-975, 2013.

[5] L. Jiang, W. Yao, Q. H. Wu, J. Y. Wen, and S. J. Cheng, "Delay-dependent stability for load frequency control with constant and time-varying delays," *IEEE Trans. Power Sys.*, 27 (2), pp. 932-941, 2012.

[6] P. Kundur, *Power System Stability and Control*, New York: McGraw-Hill, 1994.

[7] S. I. Niculescu, *Delay Effects on Stability: A Robust Control Approach*, LNCIS, vol. 269, Heidelberg: Springer-Verlag, 2001.

[8] H. Özbay, C. Bonnet, and A. R. Fioravanti,"PID controller design for fractional-order systems with time delays," *Systems and Control Letters*, 61, pp. 1823, 2012.

[9] G. J. Silva, A. Datta, and S. P. Bhattacharyya, "New results on the synthesis of PID controllers," *IEEE Trans. Automatic Control*, 47 (2), pp. 241-252, 2002.

[10] W. Tan, "Unified tuning of PID load frequency controller for power systems via IMC," *IEEE Trans. Power Sys.*, 25 (1), pp. 341-350, 2010.

[11] W. Tan,"Tuning of PID load frequency controller for power systems," *Energy Convers. Manage.*, vol. 50, no. 6, pp. 14651472, 2009.

[12] X. F. Yu, and K. Tomsovic, "Application of linear matrix inequalities for load frequency control with communication delays," *IEEE Trans. Power Sys.*, 19 (3), pp. 1508-1519, 2004.

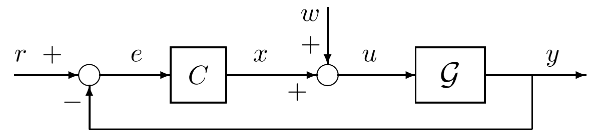

Fig. 1. The feedback system $Sys(\mathcal{G}, C)$.

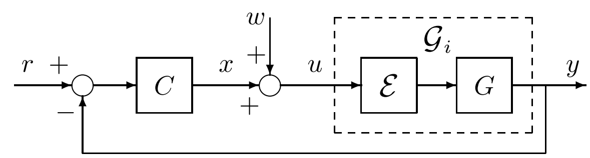

Fig. 2. The feedback system $Sys(\mathcal{G}_i, C)$ with input delay in the plant

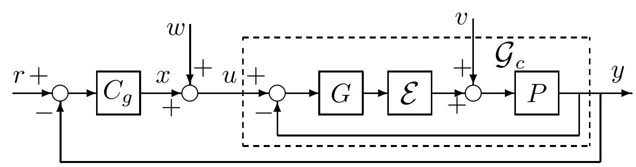

Fig. 3. The system $Sys(\mathcal{G}_c, C_g)$ with interconnected cascade delay

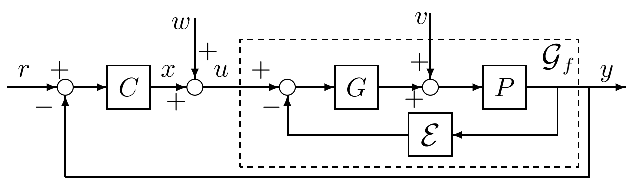

Fig. 4. The system $Sys(\mathcal{G}_f, C)$ with interconnected feedback delay

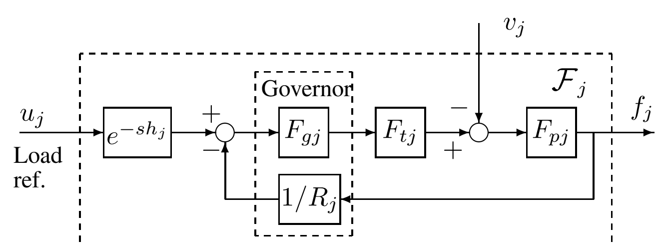

Fig. 5. Single-area power system with communication delay

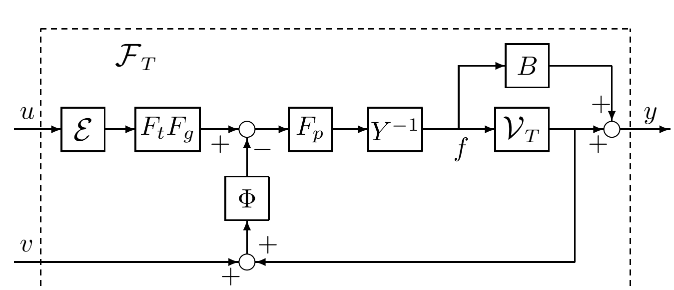

Fig. 6. Multi-area power system $\mathcal{F}_T$ with a tie-line network $\mathcal{V}_T$