1. a) A filter has a transfer function \( H_1(j\omega) = \frac{-j2\omega}{(2+j\omega)} \). What is the phase of this transfer function [or the angle \( H_1 \)] at \( \omega = 2 \) rad/s?

Phase or angle =

\[
\text{angle } H_1 = \text{angle } (-j2) - \text{angle } (2+j2) \\
= -90^\circ - \tan^{-1}\left(\frac{2}{2}\right) = -90^\circ - 45^\circ \\
= -135^\circ
\]

b) At what frequency \( \omega \) does the amplitude (or magnitude) of \( H_1 \) equal \(-6 \text{ dB} \)?

\[
\omega =
\]

\[
-6 \text{ dB} = 20 \log |H_1| \Rightarrow |H_1| = \frac{1}{2}
\]

\[
\frac{1}{2} = \left| \frac{-j2\omega}{2+j\omega} \right| =
\]

\[
= \frac{2\omega}{\sqrt{2^2 + \omega^2}}
\]

Square:

\[
\left(\frac{1}{2}\right)^2 = \frac{4\omega^2}{\omega^2 + \omega^2}
\]

\[
\Rightarrow \frac{1}{4}(\omega^2 + \omega^2) = 1 + \frac{\omega^2}{4} = 4\omega^2 \Rightarrow \omega = 0.52 \frac{\pi}{5}
\]
1 (con't):
c) What type of filter is \( H(0) = \frac{-j2\omega}{2+j\omega} \)? Check one box:

- [ ] a low-pass filter.
- [x] a high-pass filter.
- [ ] a bandpass filter.
- [ ] none of the above.
2) What are the magnitude and phase of the complex number \(-1 + j4\)?

a) magnitude or amplitude = \[ \sqrt{1^2 + 4^2} = \sqrt{17} = 4.1 \] (not in dB)

b) Phase or angle = \[ \phi = \tan^{-1} \left( \frac{4}{1} \right) = 76^\circ \]

\[ \theta = 180^\circ - 76^\circ = 104^\circ \]
3: For the signal $v(t) = (0.8 \, \text{V})\cos(1000t)$:

$v(t) \rightarrow 0.8 \, V_p$

a) What is the peak-to-peak voltage of this waveform? 

$$2(0.8 \, \text{V}) = 1.6 \, \text{V}$$

b) What is the rms voltage for this waveform?

$$\text{rms} = \frac{0.8 \, \text{V}}{\sqrt{2}} = 0.566 \, \text{V}$$
4. Find the transfer function \( H(j\omega) = \frac{V_o(j\omega)}{V_{in}(j\omega)} \) for the circuit below. (The answer should be in the form of a ratio of two complex expressions like \( 'a + jb' \), where \( a \) and/or \( b \) may be a function of \( \omega \).)

\[
H(j\omega) = \frac{R_2 + \frac{1}{j\omega C}}{R_1 + R_2 + \frac{1}{j\omega C}}
\]

\[
Z_2 = R_2 + \frac{1}{j\omega C}
\]

\[
\frac{V_o}{V_{in}} = \frac{Z_2}{Z_1 + Z_2} = \frac{R_2 + \frac{1}{j\omega C}}{R_1 + R_2 + \frac{1}{j\omega C}} = \frac{j\omega R_2 C + 1}{(R_1 + R_2) j\omega C + 1}
\]
5. A function generator in our lab has generated a voltage signal \( v(t) \). That signal is shown below, as it appears on an oscilloscope in our lab. This voltage can be expressed in the form:

\[
v(t) = A + B \sin(2\pi ft)
\]

What are \( A \), \( B \) and \( f \)?

**NOTE:** Scope settings are: 1 V/division on the vertical scale and 0.5 ms/division on the horizontal scale. Ground = 0 V is marked on the left.

\[
A = -4 \text{ V} \quad B = 1 \text{ V} \quad f = 500 \text{ Hz}
\]

\[
T = \frac{1}{f} = 4 \text{ div } \quad \text{AC pk} = 1 \text{ div}
\]

\[
T = 4 \text{ div} \times 0.5 \text{ ms/div} = 2 \text{ ms} \quad \Rightarrow f = \frac{1}{T} = 500 \text{ Hz}
\]

\[
A = DC = -4 \text{ div} \times 1 \text{ V/div} = -4 \text{ V}
\]

\[
B = \text{pk AC} = 1 \text{ div} \times 1 \text{ V/div} = 1 \text{ V}
\]
6. a) Draw the approximate Bode magnitude (or amplitude) plot \(|H| \text{ in } \text{dB} \text{ vs. } \omega \text{ on a log scale}\) for the transfer function \(H(s) = \frac{s + 10}{s}\). (Set \(s = j\omega\).) (A Bode plot using only straight-line segments is acceptable here.)

\[
|H(j\omega)| = \frac{10(1 + \frac{j\omega}{10})}{j\omega}
\]

\[20 \log |H| = 20 \log 10 + 20 \log \left| 1 + \frac{j\omega}{10} \right| - 20 \log \omega\]
Blank page

(1) $20 \log 10$

(2) $20 \log |1 + \frac{1}{s^2}|$

(3) $-20 \log \omega$

$20 \text{dB}$

$10 \text{dB}$

$0 \text{dB}$

$-20 \text{dB}$
b) Draw the approximate Bode phase (or angle) plot [angle(H) vs. \( \omega \) on a log scale] for the transfer function \( H(s) = \frac{s + 10}{s} \). (Set \( s = j\omega \).) (A Bode plot using only straight-line segments is acceptable here.)

\[
\text{angle } H = \text{angle } 10 + \text{angle} \left(1 + \frac{j\omega}{10}\right) - \text{angle}(j\omega)
\]

\[
= 0^\circ + \tan^{-1} \frac{\omega}{10} - 90^\circ
\]

↑ (A) (B)

ignore
7. In the circuits below, assume the op amp is ideal.

a) For the circuit below, what is \( V_o / V_{in} \)? [Write an expression that is a function of \( L, R \) and \( \omega \).]

\[
V_o / V_{in} = - \frac{Z_2}{Z_1} = - \frac{j\omega L}{R}
\]

b) For the circuit below, what is the output voltage, \( V_o \)? (The current source and voltage source are DC sources.)

\[
V_o =
\]

\[
V_e = 0V
\]

\[
\Rightarrow V_+ = 5V
\]

KCL at \( V_- \):

\[
1A = \frac{5V}{10\Omega} + \frac{5V - V_o}{10\Omega}
\]

\[
\Rightarrow V_o = 0V
\]