In Lab #2, you will construct simple 1st-order RL and RC filter circuits and investigate their frequency responses (amplitude and phase responses).

You will calculate the amplitude and phase responses of the filter circuits (see ‘Lab Calculations’ below).

Also, you will build and measure these quantities in lab using a sinusoidal input signal. Finally, you will plot the measured and calculated values together, answer a few questions, and turn in a report that includes: your calculations, data, plots and answers to the questions.

Each person must hand in a report.

**Background:** Assume a filter circuit has a transfer function $H(j\omega) = \frac{V_2(j\omega)}{V_1(j\omega)}$, where $V_1$ is the phasor of circuit’s input voltage and $V_2$ is the phasor of the output. (The transfer function has been or will be explained in lecture. It is also covered in your textbook.) $H(j\omega)$ is complex and can be written as

$$H(j\omega) = |H(j\omega)| e^{j\phi}$$

where $|H(j\omega)|$ is the amplitude (or magnitude) of the transfer function and $\phi = \phi(j\omega)$ is the phase of the transfer function at frequency $\omega = 2\pi f$.

If the input to the filter circuit is $v_1(t) = A_1 \cos(2\pi ft)$
and if the transfer function is $H(j\omega)$ above, then the output of the circuit will be

$$v_2(t) = A_2 \cos(2\pi ft + \phi) = A_1 |H(j\omega)| \cos(2\pi f(t - t_d)).$$

where $t_d$ is the time delay between $v_1(t)$ and $v_2(t)$, and $A_1$ and $A_2$ are positive. From the last equation, we get

$$\phi \text{ (in radians)} = -2\pi f t_d$$
$$\phi \text{ (in degrees)} = -360^\circ f t_d$$
$$|H(j\omega)| = \frac{A_2}{A_1}.$$  

Use miscellaneous parts to build the circuits; use the function generator to generate a sinusoidal input signal; use the oscilloscope to measure the signal amplitudes (which can then be used to find $|H(j\omega)|$) and to measure $t_d$ (and then calculating phase $\phi$) – see the last equation above.

Note: A breadboard is very helpful but not absolutely necessary for this lab. It will be needed in lab 3 and all other labs.

**I. Lab Calculations**

**A. Calculations for Series RL Circuit**

For the series RL circuit shown below ($R = 1k\Omega$ and $L = 100$ mH):
First, find the transfer function $H = \frac{V_2}{V_1}$ for the circuit. Then, for the following frequencies: 50 Hz, 100 Hz, 200 Hz, 400 Hz, 800 Hz, 1600 Hz, 3200 Hz, 6400 Hz, 12800 Hz, calculate the amplitude and phase (in degrees) of the transfer function.

Fill in the table on the last page of this document with your calculated values. Bring this table to lab with you; you will be recording measurements in the table and comparing them to your calculations.

**B. Calculations for Series RC Circuit #1**

For the series RC circuit shown below (R = 1kΩ and C = 1 \( \mu \)F):

Find the transfer function $H = \frac{V_2}{V_1}$ for the circuit. Calculate the amplitude and phase (in degrees) of the transfer function for the same frequencies chosen for the LR circuit above.

Fill in the table on the last page of this document with your calculated values.

**C. Calculations for Series RC Circuit #2**

Switch the positions of R and C in part B above, and calculate the amplitude and phase responses for the same frequencies as before. Fill in the table, and bring it to lab.
II. Laboratory Measurements

A. Series RL Circuit Response

Set up the signal source and measuring instruments.

**function generator**: Set its output to a 1 Vpp sinusoid at 1 kHz with 0 V DC offset. Verify the generator output on the scope. The generator output can be connected to the circuit input \([v_1(t)]\) with a BNC-to-alligator cable. (Note – The function generator reading is frequency \(f\) in Hz. Also, Vpp means Volts peak-to-peak.)

**oscilloscope (or ‘scope’)**: You will need two scope probes for this lab so you can view both the input and output signals; use two BNC-to-alligator cables for probes.

Set up the scope to view two signals at once (\(v_1\) on Channel 1, \(v_2\) on Channel 2). Set both Volt/Div scales to the same value initially and set the Time/Div scale to show about two full periods of the input. Adjust the vertical positions to have the input in the top half of the display, and the output in the bottom half of the display. Trigger the scope on the input signal.

**Power supplies**: Not needed for this lab.

**Build the circuit and instrument set-up.**

1. **Circuit**: Get a 1k\(\Omega\) resistor and a 100 mH inductor. Build the RL circuit in Fig. 1. If you don’t have a protoboard yet (you’ll definitely need one next week!), twist or clip them together so they are connected at one node.

2. **Input Signal**: Connect the function generator to the circuit with a BNC-alligator cable: Clip the black (ground) alligator clip to the ‘open’ end of the resistor (the end that is not connected to the inductor), clip the red (signal) alligator clip to the ‘open’ end of the inductor (the end that is not connected to the resistor). The function generator will produce \(v_1(t)\).

3. **Scope Connection**: Clip the Channel 1 (Ch 1) oscilloscope probe’s ground clip (black) to the bottom node of the resistor (this node will be the ground node) and the Ch 1 probe’s signal clip (red) to the input side of the inductor to allow measurement of the input \(v_1(t)\). Clip the Channel 2 probe’s ground clip to the ground node and the Ch 2 probe’s signal clip to the output side of the inductor (where the L and the R are connected). Ch 2 will measure the output \(v_2(t)\).

4. **Adjustment**: Looking at the Ch 1 waveform on the scope, adjust the function generator output to get about 1.0 Vpp at the input.

While taking measurements, keep the amplitude of the input \(v_1(t)\) roughly constant. A good function generator should maintain a constant amplitude when you only change its output frequency (but don’t count on that, especially since not all our function generators are ‘good’).
**Make the measurements.**

Make measurements at the same frequencies as your lab calculations: 50 Hz, 100 Hz, 200 Hz, 400 Hz, 800 Hz, 1600 Hz, 3200 Hz, 6400 Hz, 12800 Hz.

Measure input and output voltage amplitudes using the scope. Record each value in the table.

Measure phase shift by displaying both the input and output signals, measuring the time delay \( t_d \) between the two signals, and converting each time delay to phase:

\[
\phi = -360^\circ \cdot f t_d = -360^\circ \cdot t_d / T, \quad \text{where } T \text{ is the period of the sinusoid (} T = 1/ f). \]

The best way to measure \( t_d \) is to use the cursors on the scope, and measure the time from a zero crossing with positive slope of the input to the nearest zero crossing with positive slope of the output. Record each value of delay and then calculate the phase shift. (Note: time delay can be positive or negative, depending upon the filter. Also, it is easier and more accurate to measure the time between zero crossings than to measure the time between peaks. Zero crossings are the points on the waveform where the voltage is zero.)

**Questions related to the results:**

Is this a high-pass, low-pass, or band-pass filter?
What might cause the measured values not to match the predicted values?

**B. Series RC Circuit Response, RC Circuit #1.**

**Circuit, instruments, adjustment**

Now build the RC circuit on page 2 using a 1kΩ resistor and a 1 µF capacitor, and connect and set up the signal source and measuring instruments, as was done for the previous circuit. Repeat the input adjustment procedure above.

**Make the measurements.**

Using the same sinusoidal input signal as before, measure and record the input and amplitudes, and delay of the output for the same input frequencies as before: 50 Hz, 100 Hz, 200 Hz, 400 Hz, 800 Hz, 1600 Hz, 3200 Hz, 6400 Hz, 12800 Hz. Then calculate the phase shift. Use the same methods as for the RL circuit.

Is this a low-pass, high-pass, or band-pass filter?

**C. Series RC Circuit Response, RC Circuit #2.**

Now exchange the positions of the R and C and do the same measurements and calculations, again.
Is this a low-pass, high-pass, or band-pass filter?

III. Report

Make plots of the amplitude response $|H(j\omega)|$ (dB) and phase response (degrees) for the transfer function $H = \frac{V_2}{V_1}$ versus frequency (Hz) for each of the three circuits (use a log scale for the frequency axis or semi-log paper); make sure you have enough decades on the frequency axis. For each circuit, plot both calculated and measured values on the same graph, so you can compare the expected and measured amplitude and phase plots.

To compute the amplitude of $H$ in dB, use:

$$|H| \text{ in } dB = 20 \log_{10}[|H|].$$

(Here, $\log$ is log base 10). Hand in your calculations, measurements, plots and answers to all questions above. Be sure to have your name on each sheet. Each individual must hand in a report. Put your lab partner’s name on the front page of your report.

Parts/Additional Equipment List

1. 1 ea. 1kΩ resistor;
2. 1 ea. 100 mH inductor;
3. 1 ea. 1 μF capacitor, non-polarized;
4. 3 BNC-alligator cables
5. 2 banana-alligator cables
| LAB 2 DATA | f (Hz) | period: $T=1/f$ (sec) | Calculated $|H|$ ampl. response (Not in dB) | Calculated $|H|$ ampl. response (dB) | Calculated Phase of $H$ (deg) | Measured V1 ampl. (Vpp) | Measured V2 ampl. (Vpp) | Measured $|H| = |V2/V1|$ (dB) | Measured delay $td$ (sec) | Measured Phase of $H$ (deg) |
|------------|--------|-----------------------|---------------------------------------------|----------------------------------|-------------------------------|---------------------|---------------------|-------------------------|---------------------|--------------------------|
| RL - Part A | 50     |                       |                                             |                                  |                               |                     |                     |                         |                     |                          |
|            | 100    |                       |                                             |                                  |                               |                     |                     |                         |                     |                          |
|            | 200    |                       |                                             |                                  |                               |                     |                     |                         |                     |                          |
|            | 400    |                       |                                             |                                  |                               |                     |                     |                         |                     |                          |
|            | 800    |                       |                                             |                                  |                               |                     |                     |                         |                     |                          |
|            | 1600   |                       |                                             |                                  |                               |                     |                     |                         |                     |                          |
|            | 3200   |                       |                                             |                                  |                               |                     |                     |                         |                     |                          |
|            | 6400   |                       |                                             |                                  |                               |                     |                     |                         |                     |                          |
|            | 12800  |                       |                                             |                                  |                               |                     |                     |                         |                     |                          |
| RC #1 - Part B | 50     |                       |                                             |                                  |                               |                     |                     |                         |                     |                          |
|            | 100    |                       |                                             |                                  |                               |                     |                     |                         |                     |                          |
|            | 200    |                       |                                             |                                  |                               |                     |                     |                         |                     |                          |
|            | 400    |                       |                                             |                                  |                               |                     |                     |                         |                     |                          |
|            | 800    |                       |                                             |                                  |                               |                     |                     |                         |                     |                          |
|            | 1600   |                       |                                             |                                  |                               |                     |                     |                         |                     |                          |
|            | 3200   |                       |                                             |                                  |                               |                     |                     |                         |                     |                          |
|            | 6400   |                       |                                             |                                  |                               |                     |                     |                         |                     |                          |
|            | 12800  |                       |                                             |                                  |                               |                     |                     |                         |                     |                          |
| RC #2 - Part C | 50     |                       |                                             |                                  |                               |                     |                     |                         |                     |                          |
|            | 100    |                       |                                             |                                  |                               |                     |                     |                         |                     |                          |
|            | 200    |                       |                                             |                                  |                               |                     |                     |                         |                     |                          |
|            | 400    |                       |                                             |                                  |                               |                     |                     |                         |                     |                          |
|            | 800    |                       |                                             |                                  |                               |                     |                     |                         |                     |                          |
|            | 1600   |                       |                                             |                                  |                               |                     |                     |                         |                     |                          |
|            | 3200   |                       |                                             |                                  |                               |                     |                     |                         |                     |                          |
|            | 6400   |                       |                                             |                                  |                               |                     |                     |                         |                     |                          |
|            | 12800  |                       |                                             |                                  |                               |                     |                     |                         |                     |                          |