S to Z mapping example

EEC 212
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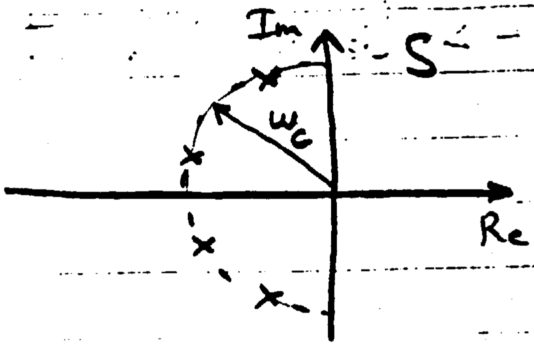
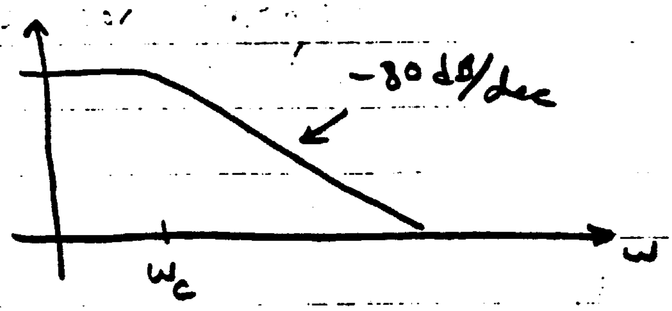
Start with Butterworth filter, 4^{th} order

Want $-3\,dB$ bandwidth $= 3\,kHz = \frac{\omega_c}{2\pi}$





$$|H_B(j\omega)| = \frac{1}{\left[1+\left(\frac{\omega}{\omega_c}\right)^{2n}\right]^{\frac{1}{2}}}$$ for n^{th} order Butterworth LPF.

$n=4$

$$\Rightarrow \quad H_B(s) = \frac{1}{\left(\frac{s}{\omega_c}\right)^4 + 2.613\left(\frac{s}{\omega_c}\right)^3 + 3.414\left(\frac{s}{\omega_c}\right)^2 + 2.613\left(\frac{s}{\omega_c}\right) + 1}$$

$$= \underbrace{\frac{1}{\left(\frac{s}{\omega_c}\right)^2 + 0.765\left(\frac{s}{\omega_c}\right) + 1}}_{H_1(s)} \cdot \underbrace{\frac{1}{\left(\frac{s}{\omega_c}\right)^2 + 1.848\left(\frac{s}{\omega_c}\right) + 1}}_{H_2(s)}$$

H_1, H_2 2^{nd} order biquads

Ex: Take $H_1(s)$, change to $H_1(z)$ using bilinear Xform: i.e. $s \rightarrow \frac{2}{T_s}\frac{z-1}{z+1}$

$$H_1(z) = \frac{1}{\left(\frac{2}{\omega_c T_s}\frac{z-1}{z+1}\right)^2 + 0.765\left(\frac{2}{\omega_c T_s}\frac{z-1}{z+1}\right) + 1}$$

$$= \frac{\left(\frac{\omega_c T_s}{2}\right)^2 (z+1)^2}{(z-1)^2 + 0.765\,\frac{\omega_c T_s}{2}(z-1)(z+1) + \left(\frac{\omega_c T_s}{2}\right)^2 (z+1)^2}$$

$$= \frac{0.0022\,(z^2 + 2z + 1)}{1.038\,z^2 - 1.9956\,z + 0.96625}$$

selecting $T_s = \frac{1}{f_s} = \frac{1}{200\,kHz}$

and with $\omega_c = 2\pi f_c = 2\pi(3\,kHz)$.

Can do the same for H_2.

Implement $H_1(z)$ & $H_2(z)$ with standard S.C. biquadratic sections.