Midterm Exam # 1 October 28, 2009

Instructions:

a) The exam is closed book, but you can use one 8½ x 11 page of notes written on both sides.

b) You may use without proof any result from either the lectures or problem sets.

Problem 1: (50 points)

Let

\[ A = \begin{bmatrix} 0 & 1 & -3 & 2 \\ 1 & 2 & -1 & 3 \\ 2 & 0 & 10 & -2 \end{bmatrix}. \]

a) Find the dimension and a basis for the four fundamental spaces of \( A \). These four spaces are the row and column spaces, and the right and left null spaces of \( A \).

b) Find all solutions of \( Ax = y \) for

(i) \( y = \begin{bmatrix} 3 \\ 5 \\ -2 \end{bmatrix} \) and (ii) \( y = \begin{bmatrix} 2 \\ 4 \\ -1 \end{bmatrix} \).
Problem 2: (50 points)

a) Find the characteristic polynomial, eigenvalues and eigenvectors of the matrix

\[
A = \begin{bmatrix}
1 & -1 & 0 \\
-1 & 2 & -1 \\
0 & -1 & 1
\end{bmatrix}.
\]

b) Use the results of part a) to find a transformation matrix $T$ such that $\Lambda = T^{-1}AT$ is a diagonal matrix.

c) Consider now the matrix

\[
B = \begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
-2 & -5 & -4
\end{bmatrix}.
\]

Find its characteristic polynomial and verify that its roots are $-1$ and $-2$. Find all eigenvectors and generalized eigenvectors of $B$.

d) What is the Jordan form of $B$?
Problem 3: (50 points)

Consider the block diagram shown in Fig. 1.

![Block diagram representation of a system.](image)

Figure 1: Block diagram representation of a system.

a) Taking the output of integrators as state variables (with the right to left labeling used in the block diagram), write the dynamic equations for this system in state-space form.

b) Eliminate all intermediate state variables to obtain a differential equation relating the input \( u(t) \) and output \( y(t) \).

c) From the point of view of input-output behavior, many different block diagrams may be equivalent in the sense of being described by the same input-output differential equation. Determine whether the two block diagrams shown in Fig. 2 are equivalent to that of part a).
Figure 2: Two other block diagrams.
Problem 4: (50 points)

Consider the signal flow graph shown in Fig. 3 below.

Figure 3: Signal flow-graph representation of a discrete-time system.

a) Obtain a discrete-time state-space representation for this signal flow graph. In order to standardize your answers, it is requested that you should label the states as indicated on the signal flow graph.

b) Find the transfer function $H(z) = Y(z)/U(z)$ by using Mason’s rule.