Connectivity of Cognitive Radio Networks:

Proximity vs. Opportunity

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Connectivity: Poisson Primary + Poisson Secondary

Connectivity: the existence of an infinite connected component almost surely.
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Existence of A Link between Two Secondary Users:

- they are within Tx range;
- they see a bidirectional opportunity.
Who Sees An Opportunity Who Doesn’t?

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**Connectivity**

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Existence of A Link between Two Secondary Users:

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The connectivity region $C$ is contiguous.

- $\lambda^*_PT(\lambda_S)$ monotonically increases with $\lambda_S$.
- $\forall (\lambda_S, \lambda_PT) \in C$, there exists a *unique* infinite connected component.
- The critical density of secondary users: $\lambda^*_S = \lambda_c(r_p)$ (*CD of homogenous networks*).
- The critical density of primary transmitters: $\overline{\lambda^*_PT} \leq \frac{\lambda_c(1)}{4 \max\{R_T^2, r_T^2\} - r_p^2}$. 

**Connectivity Region**

![Connectivity Region Graph](image-url)
Critical Density of Primary Transmitters

\[
\lambda_{PT} \text{ (per km}^2\text{)}
\]

\[
\lambda_S \text{ (per km}^2\text{)}
\]
Proof Techniques

- Contiguity and monotonicity: coupling argument

\[
\text{Poisson}(\lambda_2) = \text{Poisson}(\lambda_1) + \text{Poisson}(\lambda_2 - \lambda_1)
\]

- Uniqueness of the infinite connected component
  - Ergodic theory \(\Rightarrow\) # of infinite connected components = constant a.s.
  - Contradiction \(\Rightarrow\) # of infinite connected components \(\in\{0, 1, \infty\}\) a.s.
  - Combinatorics \(\Rightarrow\) # of infinite connected components \(\neq \infty\) a.s.
Proof Techniques

- Critical density of secondary users
Discretize the continuum model into a dependent edge-percolation model.
Proof Techniques

- Critical density of primary transmitters
  - An infinite connected component in the secondary network $\Rightarrow$ An infinite vacant component in the two Poisson Boolean models driven by the primary transmitters and receivers.
  - Sharp transitions for two-dimensional Poisson Boolean models

\[ \lambda_c \quad \lambda \]

- no infinite occupied component
- a unique infinite occupied component
- a unique infinite vacant component
- no infinite vacant component
Outer Bound on Connectivity Region

- Necessary condition for connectivity: conditional average degree $\mu > 1$.

**Proof:** construct a branching process, where $\mu = \mathbb{E}[^\#\text{offspring}]$.

- Conditional average degree $\mu$

\[
\mu = \mathbb{E}[\text{degree} \mid \text{the secondary user sees the opportunity}]
\]

\[
= (\lambda_S \pi r_p^2) \cdot \Pr\{\text{opportunity} \mid \text{one secondary user sees the opportunity}\}.
\]
Inner Bound on Connectivity Region

Sufficient condition for connectivity:

\[
1 - \exp \left( -\frac{\lambda_S r_p^2}{8} \right) \exp \left\{ -\lambda_{PT} \pi \left[ R_I^2 + r_I^2 - I(R_I, R_p, r_I) \right] \right\} > p_c,
\]

where

\[ I(r, R_p, r_I) = 2 \int_0^r t S_I(t, R_p, r_I) \frac{\pi R_p^2}{\pi R_p^2} \, dt, \]

\( S_I(t, R_p, r_I) \) is the common area of two circles with radii \( R_p \) and \( r_I \) and centered \( t \) apart, and \( p_c \) is the upper critical probability of a constructed dependent site-percolation model \( \mathcal{L} \).

**Proof:** the ergodicity of the network model and its relation with \( \mathcal{L} \).
Proximity vs. Opportunity

Increasing $p_{tx}$ leads to more neighbors but fewer opportunities.
Proximity vs. Opportunity

![Graph showing the relationship between transmission range $r_p$ and conditional average degree $\mu$.]
Proximity vs. Opportunity

\[ \lambda_{PT} \]

\( p_{tx} \)

\[ \lambda_S \]

x \( 10^{-3} \)

small power

large power
Optimal Transmission Power

To coexist with heavy traffic load: Tx power matching between primary and secondary.
Conclusion

\[ \text{Connectivity Region } \mathcal{C} \]

\[ \lambda_{PT}^*(\lambda_S) \]

\[ \lambda_{PT}^* \]

\[ \lambda_S^* \]